Mathematics Instructional Cycle Guide

4.NF.1 Equivalent Fractions

Created by Mary Lou Woods,
2014 Connecticut Dream Team teacher

CT CORE STANDARDS

This Instructional Cycle Guide relates to the following *Standards for Mathematical Content* in the *CT Core Standards for Mathematics*:

4.NF.1. Explain why a fraction *a/b* is equivalent to a fraction *(nxa)/(nxb)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

This Instructional Cycle Guide also relates to the following *Standards for Mathematical Practice* in the *CT Core Standards for Mathematics*:

Standard 1—Make Sense of Problems and Persevere in Solving Them
Standard 2—Reason Abstractly and Quantitatively
Standard 3—Construct Viable Arguments and Critique the Reasoning of Others
Standard 6—Attend to Precision

WHAT IS INCLUDED IN THIS DOCUMENT?

- A Mathematical Checkpoint to elicit evidence of student understanding and identify student understandings and misunderstandings **(p. 3)**
- A student response guide with examples of student work to support the analysis and interpretation of student work on the Mathematical Checkpoint **(pp. 4-7)**
- A follow-up lesson plan designed to use the evidence from the student work and address the student understandings and misunderstandings revealed **(pp. 8-12)**
- Supporting lesson materials **(pp. 13-18)**
- Precursory research and review of standard **4.NF.1** and assessment items that illustrate the standard **(pp. 19-21)**

HOW TO USE THIS DOCUMENT

1) Before the lesson, administer the **Joe's and Mike's Candy Bars** ***Mathematical Checkpoint*** individually to students to elicit evidence of student understanding.
2) Analyze and interpret the student work using the ***Student Response Guide***
3) Use the next steps or ***follow-up lesson plan*** to support planning and implementation of instruction to address student understandings and misunderstandings revealed by the Mathematical Checkpoint
4) Make instructional decisions based on the checks for understanding embedded in the follow-up lesson plan

MATERIALS REQUIRED

- **Fraction Bars or Cuisenaire Rods**
- **Paper Strips (6 per student of equal size)**
- **Worksheets included in the lesson**

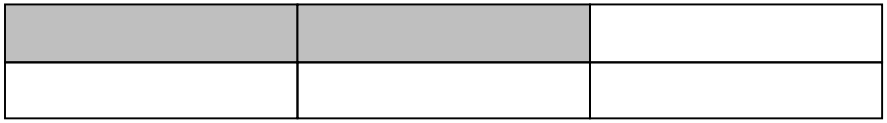
TIME NEEDED

Joe's and Mike's Candy Bars administration: **20 mins.**
Follow-Up Lesson Plan: **2-- 60 min session.**

Timings are only approximate. Exact timings will depend on the length of the instructional block and needs of the students in the class.

<table>
<tr><th colspan="3">Step 1: Elicit evidence of student understanding</th></tr>
<tr><th colspan="3">Mathematical Checkpoint</th></tr>
<tr><th>Question(s)</th><th colspan="2">Purpose</th></tr>
<tr><td rowspan="2">Joe and Mike each ate a fraction of their candy bars. The pictures below show the fraction amount each person ate.

Joe's Candy Bar
 

Mike's Candy Bar
 

Which statement is right? Explain your thinking.</td><td>CT Core Standard:</td><td>4.NF.1. Explain why a fraction a/b is equivalent to a fraction (nxa)/(nxb) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</td></tr>
<tr><td>Target question addressed by this checkpoint:</td><td>Do students understand that two fractions are equivalent if they take up the same amount of space in a region?

How do students explain their thinking to show that a unit fraction (1/b) is equivalent to another fraction (c/d)?</td></tr>
</table>

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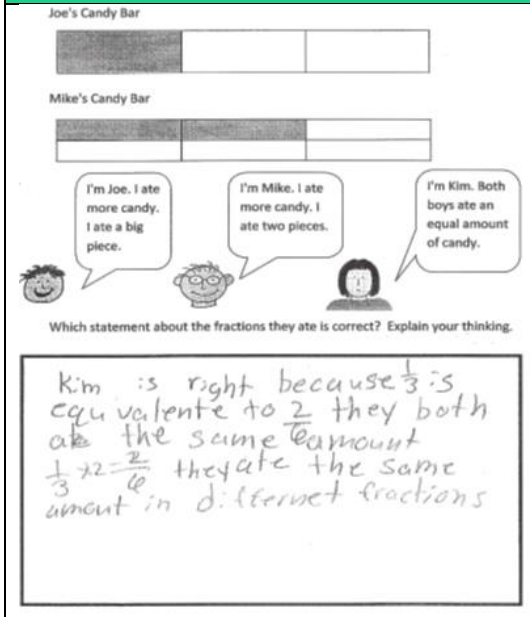
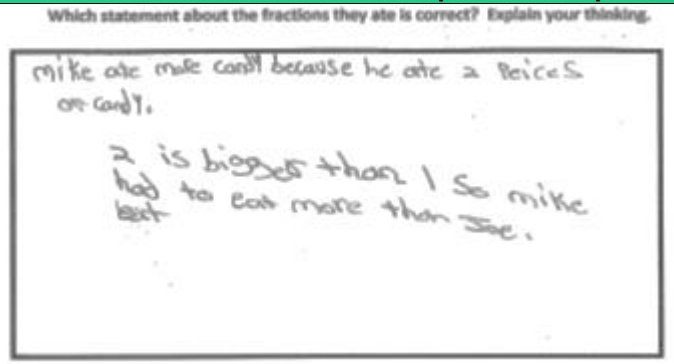
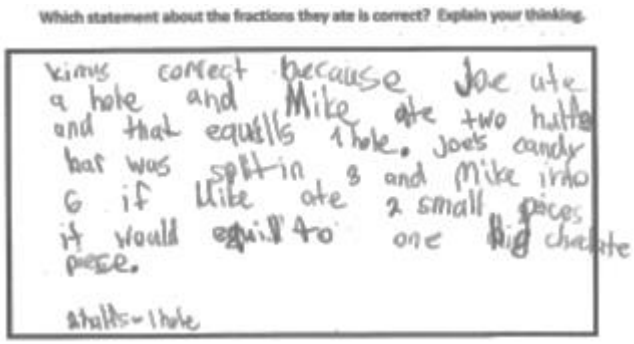
Step 2: Analyze and Interpret Student Work Student Response Guide		
Got It	**Developing**	**Getting Started**
		

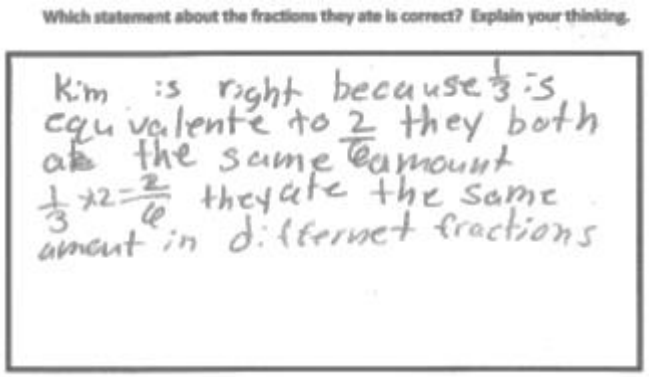
Getting Started	
Student Response Example	**Indicators**
	• Student may not understand equivalent fractions mean that the portions of the whole are equal in size to each other. • Student did not use the visual representation of the fractions to determine that these fractions are equivalent. • Student does not refer to fractional portions of a whole, but perceives each part to be one whole.
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q: What do you think this question is asking? Q: What do you notice about the size of the shaded parts? Q: What do you notice about the un-shaded parts? Q: What do you call a part of a whole? P: Give student two pieces of paper that are equal in size. Allow student to fold the paper to represent 3/3 and 6/6. Ask student to cut the pieces of paper on the folds and label the pieces with fraction names (1/3, 1/6). Ask student to put the pieces together to determine that, when together, they equal one whole.	Identify equivalent fractions using fraction models https://ctdreamteam.learnzillion.com/lessons/1731-identify-equivalent-fractions-using-fraction-models Identify equivalent fractions using fraction strips https://ctdreamteam.learnzillion.com/lessons/1733-identify-equivalent-fractions-using-fraction-strips Provide student with fraction tools (circles, rectangles, etc.) that would allow them to find equivalent fractions. Using strip paper will also allow the student to determine which fraction pieces are equivalent to each other. Students can also make their own fraction strips or use other fraction manipulatives (circles, bars, number lines) to identify equivalent fractions.

<table>
<tr><th colspan="2">Developing</th></tr>
<tr><th>Student Response Example</th><th>Indicators</th></tr>
<tr><td> 
</td><td>

Student response indicates that there is some understanding of equivalent fractions. The student used visualization to explain that 2 pieces of one bar equals one piece of another bar.
Students may refer to the two pieces shaded as halves and the one shaded piece as the whole
Student reasoning is incomplete as it does not refer to 1/3 or 2/6 and their equivalence.

</td></tr>
<tr><th>In the Moment Questions/Prompts</th><th>Closing the Loop (Interventions/Extensions)</th></tr>
<tr><td>Q: How would you describe the size of Joe’s and Mike’s candy bars?
Q: How do you know these are equivalent?
Q: How do we make equivalent fractions?</td><td>Using paper of equal size, have the student cut fractional amounts to show that each time a half is cut, the number of pieces is multiplied by two. Lead the student to understand that when adding partitions to a fraction, the size of the region has not changed but the name of each piece is different because there are now more pieces in the whole.

Recognize equivalent fractions using area models
https://ctdreamteam.learnzillion.com/lessons/616-recognize-equivalent-fractions-using-area-models</td></tr>
</table>

Got it	
Student Response Example	**Indicators**
	• Student names the two equivalent fractions. • Student justifies the equivalence by using an equation. • Student represents the idea that fractions are equivalent because they have the same amount • Student uses multiplication to demonstrate how to make equivalent fractions
In the Moment Questions/Prompts	**Closing the Loop (Interventions/Extensions)**
Q: Is there another way to prove that these two fractions are equivalent? Q: What is another fraction that is equivalent to these fractions? P: Draw a different equation to show two fractions that are equivalent.	Give an example when $\frac{1}{3}$ and $\frac{2}{6}$ would not be equivalent to each other. Fractions and Rectangles task from Illustrative Mathematics **https://www.illustrativemathematics.org/illustrations/881**

Steps 3 and 4: Act on Evidence from Student Work and Adjust Instruction	
Lesson Objective:	Generate equivalent fractions using visual fraction models and demonstrate a mathematical principle to generate equivalent fractions.
Content Standard:	**4.NF.1.** Explain why a fraction *a/b* is equivalent to a fraction *(nxa)/(nxb)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
Targeted Practice Standard :	Standard 7: Look for and make use of structure. Standard 8: Look for and express regularity in repeated reasoning.

Mathematical Goals	**Success Criteria**
Demonstrate that two fractions are equivalent by using visual fraction models of equivalent length *Understand that the principle that a/b=(nxa)/(nxb) generates equivalent fractions.*	*Use multiplication to create equivalent fractions.* *Explain how two or more fractions are equivalent using multiplication of each fractional part of the whole.*

Launch (Probe and Build Background Knowledge)

Purpose: Access and build background of equal partitioning of a whole and equivalent fractions.

True or False, $\frac{1}{2} = \frac{4}{8}$? Draw a picture to prove your answer.

Questions to guide discussion:

Share your reasoning with the class.

What model did you draw? What other models could prove your answer? (models might include circles, rectangles, bar, number line)

How are $\frac{1}{2}$ and $\frac{4}{8}$ the same? How are they different?

Instructional Task

Purpose: To generate a mathematical principle for creating equivalent fractions.

Engage(Setting up the task):

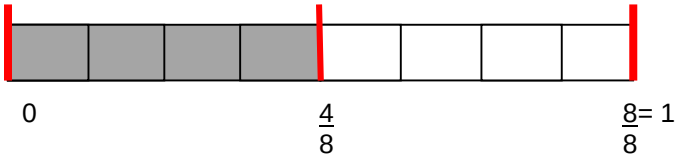
Connect the students' discussion and models in the launch to the principle for creating equivalent fractions. Using a bar diagram, shade half. Have a student partition each half to show that $\frac{4}{8}$ is equal to half.

Model writing an equation for these equivalent fractions.
When we partitioned each half into 4 smaller equal parts we created 4 times as many equal parts (write x 4 beside the denominator). We started out with 1 shaded part. When we partitioned each half into 4 smaller equal parts, we also created 4 times as many shaded parts (write x4 beside the numerator). The name for the fraction is now 4/8, but the same amount of the bar is shaded.

We can write an equation to prove this:

$$\frac{1}{2} = \frac{4 \times 1}{4 \times 2} = \frac{4}{8}$$

We multiplied each section four times. Now we have a total of 8 sections, but four of them make up one half.



Introduce the task directions

Mr. Woods is a carpenter. He uses lots of pieces of wood to make things. He is building different train tracks for his son's wooden train set. In order to do this, he will use the scrap pieces of wood he has.

He has 1 piece of oak.

He has 2 pieces of maple wood the same size.

He has 4 pieces of cherry wood the same size.

He has 8 pieces of Douglas Fir the same size.

He has 6 pieces of pine wood the same size.

He has 12 pieces of cedar wood the same size.

- *What do you notice about each of these different types of wood? (Each individual piece is the same size within that type of wood. Each amount of wood used is the same length as the piece of oak, etc.)*
- *How many pieces of cedar (and other types of woods) make up the whole?*
- *How can we write these as fractions?*
- *What are all the ways Mr. Woods could create $\frac{1}{2}$ track? $\frac{1}{4}$ track?* (See student assignment page 14)

It is important for students to demonstrate that in order for fractions to be equivalent, the wholes being compared must be equal in size, yet portioned differently.

Explore (Solving the Task)

Working with their partners, students will design train tracks for Mr. Woods's son in fractional amounts equaling halves or fourths. As students work, administer "Check for Understanding" observation questions.

Checking for Understanding

Purpose: To determine if students can demonstrate a mathematical principle to show equivalence of fractions.

- *How do you know your fractions are equivalent?*
- *What patterns do you notice?*
- *What other fractions could you show that are equivalent to $\frac{1}{2}$ and $\frac{1}{4}$?*
- *What mathematical principle can you use to demonstrate that equivalence?*

Elaborate (Discuss Task and Related Mathematical Concepts)

Students will switch partners upon completion of the train tracks to compare results. Students will record the fractional amounts equal to $\frac{1}{2}$ and $\frac{1}{4}$. After a period of discussion, students will come back as a whole group to discuss their findings.

Discussion questions:

How did you know which fractions were equal to $\frac{1}{2}$? What patterns did you notice?
How did you know which fractions were equal to $\frac{1}{4}$? What patterns did you notice?
What rule could you use to find an equivalent fraction for any other fraction?

Class Discussion

Using a multiplication chart, what patterns can you see related to $\frac{1}{2}$?

x	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

On a whiteboard or scrap paper, have the students use a bar model to demonstrate the connection between $\frac{1}{2}$ and $\frac{5}{10}$, relating this back to the patterns on the multiplication chart. Help them to see that the size of the units gets smaller, but there are more units in the whole. Relate this to the equation

$$\frac{1}{2} = \frac{1\ x\ 5}{2\ x\ 5} = \frac{5}{10} \qquad \frac{(5\ times\ as\ many\ selected\ units)}{(5\ times\ as\ many\ units\ in\ the\ whole)}$$

Have the students demonstrate connections between the multiplication chart and other equivalent fractions, such as fractions that are equivalent to $\frac{1}{4}$.

Ask students to share their rules for creating equivalent fractions.
Will this rule always work? How can you prove this rule using the models and equations?

(See http://www.math.com/tables/general/multiplytable.htm or page 15 for a multiplication table.)

Common Misunderstanding

Purpose: Students will assume that two fractions are not equivalent because one fraction's numerator and denominator are made from digits that are greater than the other fraction.

Joey says that $\frac{8}{10}$ is more than $\frac{4}{5}$ because 8 is more than 4 and 10 is more than 5. Do you agree? Show a model and an equation to explain your thinking. (see page 13)

Pose the question to students. Discuss their models and relate the equations to the work done above.

Checking for Understanding

Purpose: *This journal entry will be given to determine students' levels of understanding of how equivalent fractions are formed.*

In journals, students will answer this question:

Write an equation that will show a fraction that is equal to 3/12. How do you know this fraction is equal to 3/12?

(Students who have difficulty may benefit from the extra practice on page 16.)

Closure

Purpose: To assess students' comfort with finding equivalent fractions. After showing your work, draw:

- a smiley face  to tell that you really understand how to find an equivalent fraction to 3/12
- a straight face  to tell that you're not sure you understand how to find an equivalent fraction to 3/12
- a sad face  to tell that you need help to find an equivalent fraction to 3/12

Extension Tasks

Purpose: Students will demonstrate understanding of generating equivalent fractions by dividing the numerator and denominator by the same number. See worksheets page 17-18.

4.NF.1 Common Misunderstanding

Joey says that $\frac{8}{10}$ is more than $\frac{4}{5}$ because 8 is more than 4 and 10 is more than 5. Do you agree? Show a model and an equation to explain your thinking.

Name_____

You will make train tracks for Mr. Woods's son, Larry. Using the materials your teacher gives you, please follow these directions.

1. Make sets of train tracks that are $\frac{1}{2}$ the length of the piece of oak. How many different sets can you make? What are their fraction names? Write the fractions in the space below. Use equations to prove that they are equal to $\frac{1}{2}$.

2. Make sets of train tracks that are equivalent to $\frac{1}{4}$ the length of the piece of oak. How many different sets can you make? What are their fraction names? Write the fractions in the space below. Use equations to prove that they are equal to $\frac{1}{4}$.

3. Name another fraction that is equivalent to $\frac{1}{4}$. Prove that your fraction is equivalent with a drawing and equation.

4. What rule could you use to find an equivalent fraction for any other fraction?

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Multiplication Table

x	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

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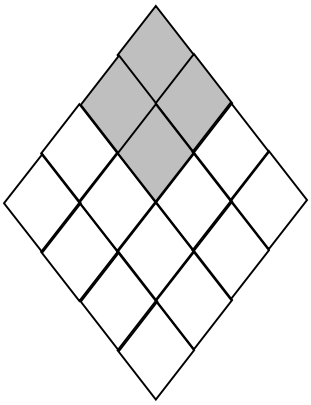
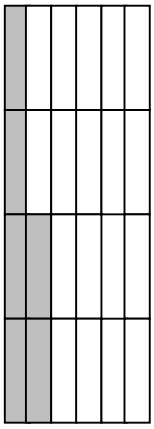
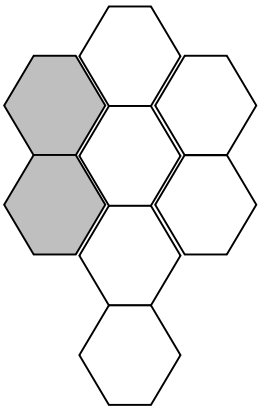
4.NF.1 Extra Practice

Joe drew three shapes. He divided each one and shaded one fourth of each shape. Help Joe explain how he knows that each of these is equivalent to one fourth by writing an equation that shows how the equivalent fractions were made to match each picture.







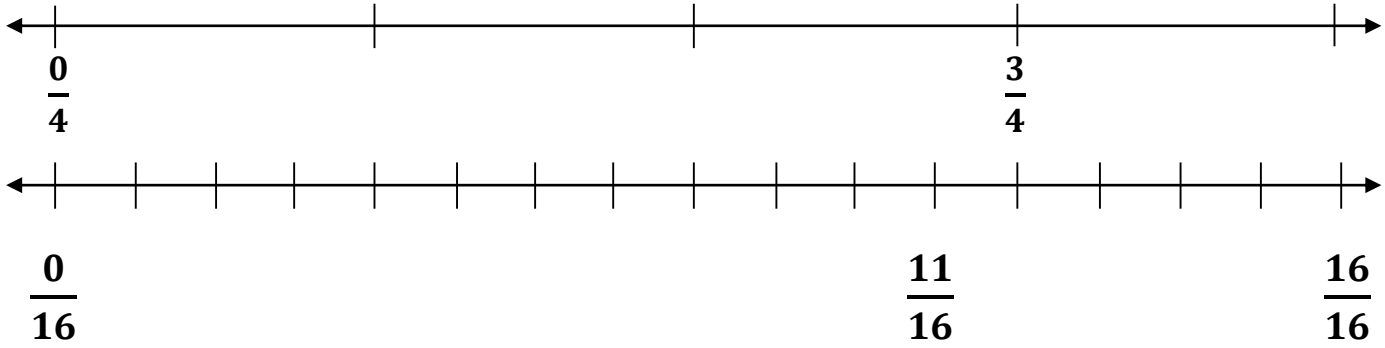
Name_____

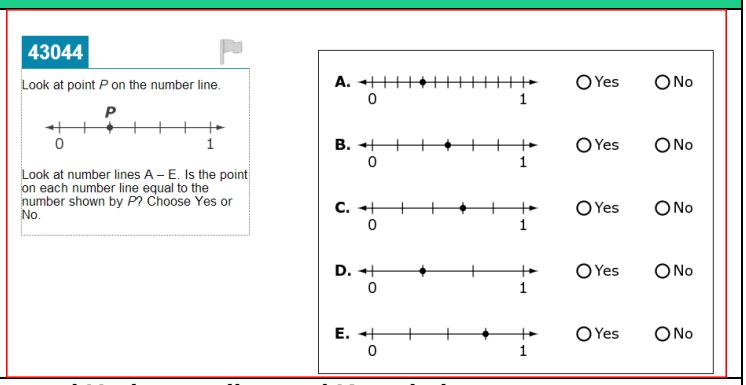
Katie says that she can make fractions that are equivalent to $\frac{10}{12}$ by using digits that are less than 12. Joe says that this is not possible because when you make equivalent fractions, you multiply the numerator and denominator by the same number and that makes digits in the numerator and denominator larger.

Who is correct? Show how you know.

Name_____ **4.NF.1 Extension Task**

Use mathematical reasoning to complete the number lines by filling in the missing fractions. Circle two pairs of equivalent fractions. Write an equation for each to show their equivalence in the boxes below.



<table>
<tr><th colspan="2">Research and review of standard</th></tr>
<tr><td>Content Standard(s):</td><td>Standard(s) for Mathematical Practice:</td></tr>
<tr><td>What standard was this item designed to assess?
4.NF.1. Explain why a fraction a/b is equivalent to a fraction (nxa)/(nxb) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</td><td>Standard 7—Look for and make use of structure –students use a unit fraction as a way to determine equivalent fractions.</td></tr>
<tr><td>Smarter Balanced Claim</td><td>Smarter Balanced Item</td></tr>
<tr><td>Claim 1 Concepts and Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.</td><td> 
</td></tr>
<tr><td>CPR Pre-Requisites
(Conceptual Understanding, Procedural Skills, and Representations)</td><td>Conceptual Understanding and Knowledge
Understand fractions as equal parts of a whole
Understand fractions on a number line
Recognize the unit fraction to determine that each different fraction is made up of equal parts as shown in the denominator
Understand that a fraction can be broken into unit fractions as a step to creating an equivalent fraction
Procedural:
Identify fractions on a number line
Representational
Represent equivalent fractions using two or more number lines
Social knowledge
Equivalent means equal in size
Fraction means part of a whole
Numerator is the amount we have
Denominator is the amount making the whole
Numerator is on the top of the fraction
Denominator is on the bottom of the fraction</td></tr>
</table>

Standards Progression		
Grade(s) below	**Target grade**	**Grade(s) above**
3.NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. 3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. 3.NF.A.2a Represent a fraction 1/b ona number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. 3.NF.A.2b Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b on the number line.	4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction (nxa)/(nxb) by using visual fraction models, with attention to how the number and size fo the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >,=, or<, and justify the conclusions, e.g., by using a visual fraction model.	5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example 2/3+5/4=8/12+15/12=23/12. (In general, a/b+c/d=(ad+bc)/bd.)

Common Misconceptions/Roadblocks

What characteristics of this problem may confuse students?

- *Students may not recognize equivalent fractions if number line examples are not used.*
- *Students may not know that the problem has more than one correct answer.*
- *Students may not know what is meant by point P on the SBAC example.*

What are the common misconceptions and undeveloped understandings students often have about the content addressed by this item and the standard it addresses?

- *Students may not understand that fractions are parts of a whole meaning the interval between 0 and 1.*
- *Students may not generalize understanding of fractions beyond using fraction pieces or circle wedges.*
- *Students may count tic marks, including 0, and therefore have an incorrect fractional amount.*
- *Students may have difficulty creating their own number line with equally spaced tic marks.*
- *Students may not realize that although the size of the whole has changed, the fraction name remains the same.*
- *Students may not realize that fractional parts can be divided again, infinitely, to make equivalent fractions.*

What overgeneralizations may students make from previous learning leading them to make false connections or conclusions?

- *Students may number a number line using whole numbers instead of fractions*
- *Students may focus on only the numerator or the denominator as a way to compare fractions .*